$$S=2\int_{AO}^{A'O}-\tfrac{1}{2}p\sin\alpha\left[\frac{xdx}{\sqrt{[-\cos(\alpha-\beta)\cos(\alpha+\beta)x^2+2px\cos\alpha\cos\beta-p^2]}}\right]$$

$$=-p\sin\alpha\left[\frac{\sqrt{[-\cos(\alpha-\beta)\cos(\alpha+\beta)x^2+2px\cos\alpha\cos\beta-p^2]}}{-\cos(\alpha-\beta)\cos(\alpha+\beta)}\right.$$

$$+\frac{p\cos\alpha\cos\beta}{\cos(\alpha-\beta)\cos(\alpha+\beta)}\left(\frac{1}{\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}\right.$$

$$\left.\left.\times\sin^{-1}\left(\frac{\cos(\alpha-\beta)\cos(\alpha+\beta)x-p\cos\alpha\cos\beta}{p\sin\alpha\sin\beta}\right)\right]_{AO}^{A'O}$$

$$=-\frac{p^2\sin\alpha\cos\alpha\cos\beta}{\cos(\alpha-\beta)\cos(\alpha+\beta)\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}[\tfrac{1}{2}\pi-\tfrac{3}{2}\pi]$$

$$=\frac{p^2\sin\alpha\cos\alpha\cos\beta\,\pi}{\cos(\alpha-\beta)\cos(\alpha+\beta)\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}=\frac{A'O.A\,O\,\pi\sin\alpha\cos\alpha\cos\beta}{\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}.$$

Since $A'O\cos^2(\alpha+\beta)=AO\cos(\alpha-\beta)$, $\dfrac{A'O}{A\,O}=\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$.

$$\frac{A'O+AO}{AO}=\frac{2\cos\alpha\cos\beta}{\cos(\alpha+\beta)}.\quad\text{Also}\ \frac{A'O+AO}{A'O}=\frac{2\cos\alpha\cos\beta}{\cos(\alpha-\beta)}.$$

$$\therefore\frac{(A'O+A\,O)^2}{A'O.AO}=\frac{4\cos^2\alpha\cos^2\beta}{\cos(\alpha-\beta)\cos(\alpha+\beta)},\ \text{or}\ \frac{\cos\alpha\cos\beta}{\sqrt{[\cos(\alpha-\beta)\cos(\alpha+\beta)]}}$$

$$=\tfrac{1}{2}\frac{A'O+AO}{\sqrt{(A'O.AO)}}.$$

$$\therefore S=\tfrac{1}{2}\pi\sqrt{(A'O.AO)}\,(A'O+AO)\sin\alpha.$$

Dr. G. B. M. Zerr sent in two very simple solutions but not by projecting the convex surface on the plane of the elliptic base. Professor Scheffer sent in a solution similar to Dr. Zerr's. As the problem presents no difficulty when referred to rectangular axis with axis of the cone as one of the axes of coordinates, these solutions are omitted. The rectangular equation of the cone referred to planes $A'OA$, APA', and a plane through OK perpendicular to $A'OA$ is $y^2=\sec^2\alpha[(p-z)\sin(\alpha+\beta)+x\cos(\alpha+\beta)][(p-z)\sin(\alpha-\beta)-x\cos(\alpha-\beta)]$.

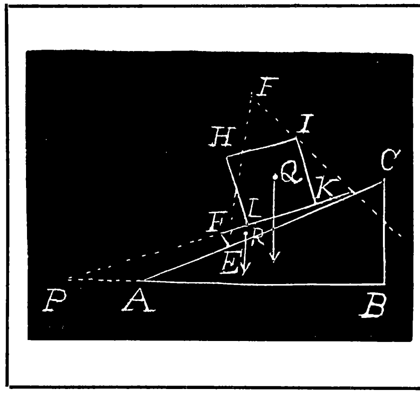
MECHANICS.

194. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

A body has a plane face resting on a rough wedge. The wedge is on a rough inclined plane, thick end down and thin edge horizontal. Find the condition that the body will slide down the wedge with constant acceleration, the wedge not slipping the while. Discuss the case in which the angle of friction for wedge and plane is greater than the angle of inclination of the plane.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va.

Constant acceleration is a force that produces a constant increase in velocity. Gravity is such a force.



Let $\angle CBA=\beta$, the angle of inclination of the plane; $\angle FDE=\delta$, the angle of the wedge. Let ν=coefficient of friction between wedge FDE and body $HIKL$.

If $\tan DPB=\tan(\beta-\delta)>\nu$, the constant acceleration is

$g[\sin(\beta-\delta)-\nu\cos(\beta-\delta)]$.

If $\angle FDE>\angle CAB$, then if $\tan(\delta-\beta)>\nu$, the constant acceleration is $g[\sin(\delta-\beta)-\nu\cos(\delta-\beta)]$.

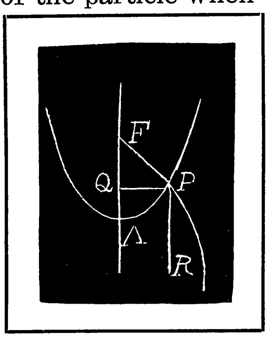
Hence, the condition for constant acceleration of the body on the wedge, the wedge not slipping, is that the coefficient of friction between body and wedge be less than the tangent of the difference of the angles of plane and wedge or wedge and plane. This covers the case for angle of friction for wedge and plane greater than angle of inclination of plane. The opposite case would form an interesting problem.

195. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Particles slide from rest at the focus of a parabola, whose axis is vertical, down radius vectors, and are then allowed to move freely. Find the locus of the foci of their subsequent paths.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let PF be a radius vector$=r$, and $\angle PFA=\theta$. Let v be the velocity of the particle when leaving P. Draw PR parallel to AF. We find with reference to PR as the axis of abscissa, and P as origin of orthogonal co-ordinates, the equation



$$x=\frac{\cos\theta}{\sin\theta}y+\frac{1}{2v^2\sin^2\theta}y^2;$$

but $v^2=2gr\cos\theta$, and $r=\dfrac{a}{\cos^2\frac{1}{2}\theta}$. $\therefore v^2=\dfrac{2ag\cos\theta}{\cos^2\frac{1}{2}\theta}$.

$\therefore$ Equation of curve

$$y^2+\frac{4a\sin\theta\cos^2\theta}{\cos^2\frac{1}{2}\theta}y=\frac{4a\cos\theta\sin^2\theta}{\cos^2\frac{1}{2}\theta}x.$$

From this equation it follows that the co-ordinates of the vertex of this